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In: Methodika, (1990), 4, 1-23.

Reprint 59/1990

MAX-PLANCK-INSTITUTE FOR PSYCHOLOGICAL RESEARCH

Leopoldstrasse 24, D-8000 Munich 40, F.R.G.

Methodika
1990, Vol. IV, pp. 1–23

The Measurement of Individual Consistency*)

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Abstract: Correlations are often falsely interpreted as measures of consistency at the individual level. This misinterpretation is a special case of the general tendency among psychologists to confuse relations obtained within aggregates of individuals with relations within individuals. In the case of correlations, this tendency is particularly strong because of a lack of measures that tap the consistency between interindividual differences at the level of the single case. A coefficient of *individual consistency* is proposed which is consistent with the correlation at the aggregate level insofar as its mean is identical with the correlation. Statistical properties of the coefficient are delineated and illustrated with Monte-Carlo studies for bivariate normal distributions. A transformation is suggested that normalizes the skewed distribution of the individual stabilities for these distributions. Some applications of these coefficients are illustrated with data on the longitudinal stability of children's social behavior.

Key words: Consistency, stability, correlation, moderator variable

1. Introduction

Since the times of William Stern (1911) it is common practice in differential psychology to measure linear relations among interindividual differences by correlations (e.g., their consistency across measures, time, or situations). These correlations are measures of consistency at the level of aggregates of individuals. Thus, they may be considered to tap the *mean individual consistency of interindividual differences* in a sample of persons. In fact, the correlation r_{XY} between X and Y is a linear function of the mean squared differences between the *z*-scores of X, Y:

$$\frac{1}{n} \cdot \sum (z_X - z_Y)^2 = 2(1 - r_{XY}). \tag{1}$$

*) I am indebted to Rolf Steyer, University of Trier, F.R.G., Heinrich Wottawa, University of Bochum, F.R.G., and an anonymous reviewer for their suggestions concerning some of the proofs.

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Hence the squared z-score difference of a person tells us something about the person's consistency in the sample: the smaller these differences are, the higher is the person's consistency between X and Y. Differences between the individual consistencies of persons in the sample accordingly inform us about the *differential consistency* in the sample; for example, the variance of the individual consistencies is a measure of the mean differential consistency in the sample.

Because the correlation is essentially the mean of some index of individual consistency, it shares all the problems that affect measures of aggregates such as means or variances. Repeatedly, but without apparent success, it has been pointed out that it is not possible to infer relations at the individual level from relations found at the level of aggregates if the *functional homogeneity* of the individuals is not guaranteed (cf., e.g, Lewin, 1931; Bakan, 1954; Thorngate, 1986).

To choose a popular example, a highly significant mean improvement of emotional adjustment after psychotherapy may be perfectly compatible with an actual non-improvement of the majority of the clients studied: a few improved very much, thereby dominating the mean, whereas most clients did not improve. Because the subjects lacked homogeneity of improvement, the result obtained for the aggregate level cannot be generalized to the individual level.

The same reasoning applies to correlations as well. This has less often been recognized. Valsiner (1986) showed that undergraduates in psychology, doctors of psychology, and the authors of *Child Development* have a strong tendency to interpret correlational findings as if they would tell us something about relations at the individual level. For example, a one-year consistency of 0.40 found for interindividual differences in the observed rate of shy behavior during free-play in preschool would be typically interpreted as follows: "(The) children showed a medium consistency of (interindividual differences in) shyness over a one-year period". As Valsiner (1986) has demonstrated, such an interpretation is ambiguous at best; most would misunderstand it by assuming that nearly all, or at least the majority of children would show a medium consistency of their individual rate of shy behavior over the one-year period.

Again, the subjects of a sample must be *functionally homogeneous* in regard to their individual consistencies if the size of a correlation in the sample can be generalized to the individuals of the sample. In the example from psychotherapy research, the functional nonhomogeneity of the clients could be easily detected by inspecting the individual change scores. What should we look for when we want to examine a bivariate distribution for *differential consistency?* What is the coefficient of *individual consistency* in this case? Surprisingly, there do not seem to exist well-known coefficients of individual consistency which could be used for this purpose.

Where differential consistency has been studied, the analysis nearly always has been restricted again to the aggregate level by applying the *moderator variable approach* introduced by Saunders (1956): the effect of a moderator variable on the consistency of the interindividual differences in another variable is evaluated within a hierarchical multiple regression model. For example, the moderating effect of self-rated consistency in some trait on the cross-situational consistency of observed behavioral indicators of this trait has been studied (cf. Paunonen & Jackson, 1985, for a review).

This approach seems well suited for testing hypothesized moderating effects of some variable on the consistency of interindividual differences in another variable. However, before it comes to hypothesis testing, one often would like to have a *description* of differential consistency in the first place: the amount of differential consistency in terms of the variance of the individual consistencies and their distributional characteristics (e.g., a bimodal distribution of the individual consistencies would suggest important subgroup differences in consistency). The moderator variable approach does not provide this descriptive information. On the other hand, the individual consistencies can be used to study the moderating effect of some variable simply by correlating this variable with the individual consistencies.

There appears to exist only one approach in the literature to measure individual consistencies. Ghiselli proposed to regard the absolute value of each individual's *residual score in the regression equation*

$$|\hat{y} - y| \qquad (2)$$

as a measure of each person's "individual predictability" and to predict these scores by external variables ("prediction of predictability"; Ghiselli, 1960, 1963). However, this approach did not prove to be very successful, mainly because the predictors found were quite specific to the criterion variables and could not be cross-validated. For these reasons, Ghiselli's approach was not pursued further although some authorities still regard it as an interesting one (cf. Wiggins, 1973, chap. 2, and Paunonen & Jackson, 1985, footnote 7).

Besides this apparent failure of Ghiselli's approach to yield stable results in empirical applications, there seems to be a more fundamental problem involved in his approach. Ghiselli chose each person's deviation from the regression line as a measure of individual consistency. Thus, he compared each person's change with the change expected by the "regression to the mean effect". It is a widespread belief among psychologists that the regression to the mean is some kind of "natural law" in psychological data sets. As Rogosa, Brandt and Zimowski (1982) and others pointed out, this belief is a myth.

A regression to the mean effect may or may not occur depending on the variable observed; it is certainly to be expected only if all inconsistency at the aggregate level (and hence all differential consistency) were just random. If there are psychologically meaningful differences among the individual consistencies, differential consistency is not random, and correcting for a falsely expected regression to the mean obscures the interpretation of the differential consistency which remains. It is mainly for this reason why I do not find Ghiselli's approach viable. Another reason is that absolute deviation scores are not consistent with the mean, the variance, and the correlation (they are consistent with the median; cf. Hays, 1981, p. 164–165).

In what follows, I will propose a coefficient of individual consistency that allows an analysis of the consistency of interindividual differences at the level of individual subjects. Because the coefficient is consistent with the correlation at the aggregate level, it makes the notion explicit that the correlation is essentially the sample mean of individual consistency scores. More importantly, the coefficient provides information beyond that mean, i.e., beyond the correlation: distributional characteristics of the individual consistencies, such as their variance, or a possible multimodality of the distribution of these consistencies, which would suggest that subgroups with different mean consistency can be distinguished.

2. Method

2.1 Desired properties of individual consistencies

A coefficient of individual consistency should have the following four properties. First, it should be invariant in linear transformations of the two variables to be compared, i.e. it should be possible to define the coefficient solely by the z-transforms of the two variables. Since the coefficient should be consistent with the correlation, this assumption is necessary.

Second, the coefficient should be a strictly decreasing function of the absolute z-score-difference of the two variables to be compared for consistency. Thus, the individual consistency of a person should be at maximum when the person has the same z-score in both variables, and it should become smaller, as the absolute difference between the two z-scores increases. This makes sense when the consistency of the scores of a person is defined at the *individual level.*

Third, the coefficient of individual consistency should be a linear function of the *squared* differences of the z-scores of the two variables. This assumption makes the coefficient commensurate with variance and correlation, which also depend upon squared differences (and not absolute differences which are consistent with the median; cf. Hays, 1981, p. 164–165).

Fourth, the sample (or the population) mean of the coefficient of individual consistency should be identical with the correlation. Since the third proposition includes the first two, it suffices to consider only the third and the fourth postulate.

2.2 Individual consistency as a random variable

In this section, individual consistency is introduced at the most general level possible as a *random variable* which is defined by any two random variables X, Y on the same probability space Ω which have existing moments of degree 1–4. This very general approach makes it easier to prove various properties of the individual consistencies. For a better understanding of the meaning of the following Theorems for psychological applications, one may conceive the probability space Ω to consist of a finite population of persons. Two random variables X, Y assign to each person p latent scores X(p), Y(p) with a certain probability; then, the expectations of X, Y exist and are the population means of X, Y.

The proofs of the following Theorems repeatedly make use of certain well-known properties of Z-transformed variables. These properties are summarized in the following Lemma.

Lemma. Let $X, Y: \Omega \to \mathbb{R}$ be random variables with existing expectations E(X), E(Y), variances $\sigma^2(X)$, $\sigma^2(Y)$, third moments $E(X^3)$, $E(Y^3)$, correlation ϱ_{XY}, and Z-transformation Z_X, Z_Y (e.g., $Z_X = (X - E(X))/\sigma(X)$). Then the following propositions hold (cf., e.g., Hays, 1981):

(1) $E(Z_X) = 0$.

(2) $E(Z_X^2) = \sigma^2(X) = 1$.

(3) $E(Z_X^3) = 0$ if X is symmetrically distributed.

(4) $E(Z_X Z_Y) = \varrho_{XY}$

(5) $E((Z_X - Z_Y)^2) = \sigma^2(Z_X - Z_Y) = 2(1 - \varrho_{XY})$.

(6) The linear regression equation $Z_Y = \varrho_{XY} Z_X + e$ holds where $e: \Omega: \to \mathbb{R}$ is the error variable with $E(e) = 0$ and $E(Z_X e) = E(Z_Y e) = 1 - \varrho_{XY}^2$ and $E(e^2) = 1 - \varrho^2{}_{XY}$. If Z_Y is not nonlinearly stochastically dependent upon Z_X, i.e. if $E(Z_Y | Z_X) = \varrho_{XY} Z_X$ for the conditional expectation of Z_Y under Z_X, then $E(Z_X^2 e) = E(Z_X e^2) = 0$.

The first Theorem proves that there is one and only one variable satisfying the four desired properties for individual consistency coefficients outlined in the preceding section.

Theorem 1. Let X, Y: $\Omega \to \mathbb{R}$ be two random variables with correlation ϱ_{XY}. Then there is one and only one random variable I_{XY} which satisfies the following three conditions:

(1) I_{XY} is independent of ϱ_{XY}.

(2) I_{XY} is a linear function of $(Z_X - Z_Y)^2$.

(3) $E(I_{XY}) = \varrho_{XY}$.

It is $I_{XY} = 1 - \frac{(Z_X - Z_Y)^2}{2}$ ("individual consistency of X, Y").

Proof. $E(I_{XY}) = 1 - \frac{1}{2} \cdot E((Z_X - Z_Y)^2) = 1 - (1 - \varrho_{XY}) = \varrho_{XY}$ according to Lemma (5). Let $J_{XY}: \Omega \to \mathbb{R}$ be a random variable satisfying (1)–(3). Hence there exist $a, b \in \mathbb{R}, b \neq 0$, with $\varrho_{XY} = E(J_{XY}) = E(a + b(Z_X - Z_Y)^2) = a + 2b(1 - \varrho_{XY})$ according to Lemma (5). For $\varrho_{XY} = 1$ it follows that $1 = a$; thus, for $\varrho_{XY} = 0$ it follows that $0 = 1 + 2b$, i.e., $b = -\frac{1}{2}$. Hence $J_{XY} = I_{XY}$.

The following Theorem proofs some basic properties of the individual consistency I_{XY} which hold without any particular assumption about X, Y.

Theorem 2. Let X, Y: $\Omega \to \mathbb{R}$ be two random variables with correlation ϱ_{XY} and individual consistency I_{XY}. Then the following propositions hold:

(1) For all $p \in \Omega$,
 (a) $I_{XY}(p) \leqq 1$.
 (b) $I_{XY}(p) = 1$ if and only if $Z_X(p) = Z_Y(p)$.
 (c) $I_{XY}(p) = 0$ if and only if $|Z_X(p) - Z_Y(p)| = \sqrt{2}$.

(2) $\sigma^2(I_{XY}) = \frac{1}{4} \cdot E((Z_X - Z_Y)^4) - (1 - \varrho_{XY})^2$.

(3) $\varrho_{XI_{XY}} = -\frac{1}{2\sigma(I_{XY})} \cdot E(Z_X(Z_X - Z_Y)^2)$ for $\varrho_{XY} \neq 1$.

(4) With decreasing correlation ϱ_{XY}, the individual consistency I_{XY} shows an increasing quadratic stochastic dependence upon X. For $\varrho_{XY} = -1$ it is $I_{XY} = 1 - 2\left(Z_X - \frac{1}{2}e\right)^2$.

Proof.

(1) is trivial.

(2) $$\sigma^2(I_{XY}) = E(I_{XY}^2) - E(I_{XY})^2$$
$$= E\left(\left(1 - \frac{(Z_X - Z_Y)^2}{2}\right)^2\right) - \varrho_{XY}^2$$
$$= E\left(1 - (Z_X - Z_Y)^2 + \frac{(Z_X - Z_Y)^4}{4}\right) - \varrho_{XY}^2$$
$$= 1 - 2(1 - \varrho_{XY}) + \frac{1}{4} \cdot E((Z_X - Z_Y)^4) - \varrho_{XY}^2$$
according to Lemma (5)
$$= \frac{1}{4} \cdot E((Z_X - Z_Y)^4) - (1 - 2\varrho_{XY} + \varrho_{XY}^2)$$
$$= \frac{1}{4} \cdot E((Z_X - Z_Y)^4) - (1 - \varrho_{XY})^2.$$

(3) For $\varrho_{XY} \neq 1$,
$$\varrho_{XI_{XY}} = \frac{1}{\sigma(I_{XY})} \cdot COV(Z_X, I_{XY}) \quad \text{for the covariance COV}$$
$$= \frac{1}{\sigma(I_{XY})} \cdot COV\left(Z_X, 1 - \frac{(Z_X - Z_Y)^2}{2}\right)$$
$$= -\frac{1}{\sigma(I_{XY})} \cdot \frac{1}{2} \cdot COV(Z_X, (Z_X - Z_Y)^2)$$
$$= -\frac{1}{2\sigma(I_{XY})} \cdot [E(Z_X(Z_X - Z_Y)^2) - E(Z_X) \cdot E((Z_X - Z_Y)^2)]$$
$$= -\frac{1}{2\sigma(I_{XY})} \cdot E(Z_X(Z_X - Z_Y)^2).$$

(4) According to Lemma (6),
$$I_{XY} = 1 - \frac{(Z_X - Z_Y)^2}{2} = 1 - \frac{1}{2} \cdot (Z_X - \varrho_{XY} Z_X - e)^2$$
$$= 1 - \frac{1}{2} \cdot ((1 - \varrho_{XY}) Z_X - e)^2$$
$$= 1 - \frac{1}{2} \cdot ((1 - \varrho_{XY})^2 Z_X^2 - 2(1 - \varrho_{XY}) Z_X e + e^2).$$

For $\varrho_{XY} = 1$ it is $I_{XY} = 1 - \frac{1}{2} e^2$. For $\varrho_{XY} < 1$, $(1 - \varrho_{XY}^2) > 0$, i.e. I_{XY} is quadratically stochastically dependent upon X. Since for $\varrho_1 < \varrho_2$ it follows that $(1-\varrho_1)^2 > (1-\varrho_2)^2$, this dependence increases with decreasing ϱ_{XY}.

For $\varrho_{XY} = -1$, $I_{XY} = 1 - \frac{(Z_X + Z_X - e)^2}{2} = 1 - 2\left(Z_X - \frac{1}{2}e\right)^2$.

Theorem 2 states that there may be a nonzero correlation between X, Y and their individual consistency. The next Theorem provides sufficient conditions for this correlation to be zero:

Theorem 3. Let $X, Y: \Omega \to \mathbb{R}$ be random variables with correlation $\varrho_{XY} \neq 1$ and individual consistency I_{XY}. If Y is not nonlinearly stochastically dependent upon X, $\varrho_{XI_{XY}}$ is a function of the asymmetry of X:

$$\varrho_{XI_{XY}} = -\frac{(1-\varrho_{XY})^2}{2\sigma(I_{XY})} \cdot E(Z_X^3).$$

If, in addition, X is symmetric, then $\varrho_{XI_{XY}} = 0$.

Proof. If Y is not nonlinearly stochastically dependent upon X, the same is true for Z_Y, Z_X. From Lemma (6) it follows that

$$\begin{aligned} E(Z_X(Z_X - Z_Y)^2) &= E(Z_X^3) - 2E(Z_X^2 Z_Y) + E(Z_X Z_Y^2) \\ &= E(Z_X^3) - 2E(Z_X^2(\varrho_{XY} Z_X + e)) + E(Z_X(\varrho_{XY} Z_X + e)^2) \\ &= E(Z_X^3) - 2\varrho_{XY} E(Z_X^3) + \varrho_{XY}^2 E(Z_X^3) \\ &= E(Z_X^3)(1 - 2\varrho_{XY} + \varrho_{XY}^2) \\ &= E(Z_X^3)(1-\varrho_{XY})^2. \end{aligned}$$

Applying this equation to Theorem 2 (3) yields the proof. If X is symmetric, then $E(Z_X^3) = 0$ according to Lemma (3), hence $\varrho_{XI_{XY}} = 0$.

If X, Y are bivariate normally distributed, they fulfill both conditions stated in Theorem 3. Furthermore, the distribution of the individual consistency has a well-known form in this case, and its variance can be expressed very simply:

Theorem 4. Let $X, Y: \Omega \to \mathbb{R}$ be bivariate normally distributed random variables with correlation ϱ_{XY} and individual consistency I_{XY}. Then the following propositions hold:

(1) $\varrho_{XI_{XY}} = 0$.

(2) For $\varrho_{XY} \neq 1$, I_{XY} is a linear function of a random variable with a χ^2-distribution with one degree of freedom:

$$I_{XY} = 1 - (1 - \varrho_{XY}^2) \cdot \chi^2_{(1)}.$$

(3) $\sigma^2(I_{XY}) = 2(1-\varrho_{XY})^2$.

(4) I_{XY} is strongly skewed to the left with skewness $-\sqrt{8}$ and excess 12.

Proof.

(1) Since X, Y have a bivariate normal distribution, they are symmetrically distributed and show no nonlinear stochastic dependence (cf., e.g., Hays, 1981, p. 463). According to Theorem 3, it is $\varrho_{XI_{XY}} = 0$.

(2) Since X, Y are bivariate normally distributed, their difference $Z_X - Z_Y$ is normally distributed (cf., e.g., Rao, 1965, p. 438). Since $E(Z_X - Z_Y) = 0$ and $E((Z_X - Z_Y)^2) = 2(1 - \varrho_{XY})$ according to Lemma (5), the random variable $\frac{Z_X - Z_Y}{\sqrt{2(1 - \varrho_{XY})}}$ has a standard normal distribution. Since the square of this variable has a $\chi^2_{(1)}$-distribution by definition, $\frac{(Z_X - Z_Y)^2}{2(1 - \varrho_{XY})} = \chi^2_{(1)}$.

(3) The variance of $\chi^2_{(1)}$ is 2. Thus, $\sigma^2(I_{XY}) = (1 - \varrho_{XY})^2 \cdot 2$.

(4) The skewness of $\chi^2_{(1)}$ is $\sqrt{8}$, its excess is 12 (cf., e.g., Abramowitz & Stegun, 1972). Since the skewness of a variable is invariant against linear transformations (except for sign change), and the excess is fully invariant against linear transformations, proposition (4) follows directly from the properties of the $\chi^2_{(1)}$-distribution.

Theorem 4 points to a problem of the individual consistency scores as defined in this section. If the variables X, Y perfectly meet the requirements for the correlation as a measure of stochastic relation, the individual consistencies are strongly skewed. Thus, it would be misleading to apply linear analyses to them – for example, computing difference scores or variances in order to analyze the data for differential stability.

Therefore, it seems necessary to transform the individual consistencies in such a way as to obtain a fairly normal distribution. The following conjecture proposes a solution to this problem:

Conjecture. Let X, Y: $\Omega \to \mathbb{R}$ be bivariate normally distributed random variables and I_{XY} their individual consistency. Then the following transformation T: $\mathbb{R} \to \mathbb{R}$ leads to a fairly normal distribution of the individual consistencies:

$$TI_{XY} = \begin{cases} \frac{1}{2} \cdot \ln\left[\frac{1.001 + I_{XY}}{1.001 - I_{XY}}\right] & \text{for } 0 \leqq I_{XY}(p) \leqq 1 \quad (p \in \Omega) \\ \ln\left[\frac{1}{1 - I_{XY}}\right] & \text{for } I_{XY}(p) < 0. \end{cases}$$

It is $T(0) = 0$ and $T(1) = 3.80$.

Remark. The transformation T is nearly identical with Fisher's *Z*-transformation for correlations if $I_{XY}(p) \geqq 0$, except for the constant 1.001 instead of 1. This slight deviation is necessary because $I_{XY}(p)$ may equal one in which case Fisher's *Z* is not defined. If $I_{XY}(p) < 0$, T must be defined differently because, contrary to correlations, I_{XY} is not distributed symmetrically around zero.

While simple computation suffices to proof the formulas for T(0), T(1), an algebraic proof of the rest of the conjecture is not easy. Instead, the normalizing property of the transformation T for bivariate normal distributions will be demonstrated in the following section with Monte-Carlo studies.

2.3. Monte-Carlo studies of individual consistencies

If X, Z are stochastically independent standard normal distributions, the variable

$$Y(r) = rX + \sqrt{1 - r^2} \cdot Z \tag{3}$$

is also a standard normal distribution. Furthermore, for each fixed value of X or Z, the respective conditional distribution is normally distributed. Thus, X, Y(r) are bivariate normally distributed with correlation r. For a given $-1 \leqq r \leqq 1$, this allows to generate approximately bivariate normal distributions with correlation r by combining two independently generated, approximately normally distributed, *z*-transformed sequences of random numbers.

For each level of $r = 0.8 - k \cdot 0.2$ $(k = 0, \ldots, 9)$, 100 different pairs of approximately standard bivariate normal distributions $X_i, Y(r)_i$ $(i = 1, \ldots, 100)$ of 100 random numbers with correlation r were generated with the function RANNOR of the Statistical Analysis System (SAS). For each pair, various descriptive indices (including the individual consistencies I_{XY} between X, Y(r)) were computed as well as the correlation between X and I_{XY} and the transformed scores TI_{XY}. Each of these descriptive indices was then averaged over the 100 pairs of variables $X_i, Y(r)_i$ for each level of r. Table 1 contains the results of these analyses.

Figure 1 shows the distribution of the individual consistencies as well as the distribution of their transformation for the pooled 100 Monte-Carlo studies (i.e., 10000 values) for five different levels of correlation r.

The effectiveness of the transformation T in normalizing the distribution of the individual consistencies can be seen from Figure 2. The more the individual consistencies approach the maximum score of one, the more they are differentiated by the transformation T. On the other hand, the more the individual consistencies are negative, the more they are dampened by T. This last property of T has the advantage that outliers in the distribution of the individual consistencies are made less extreme.

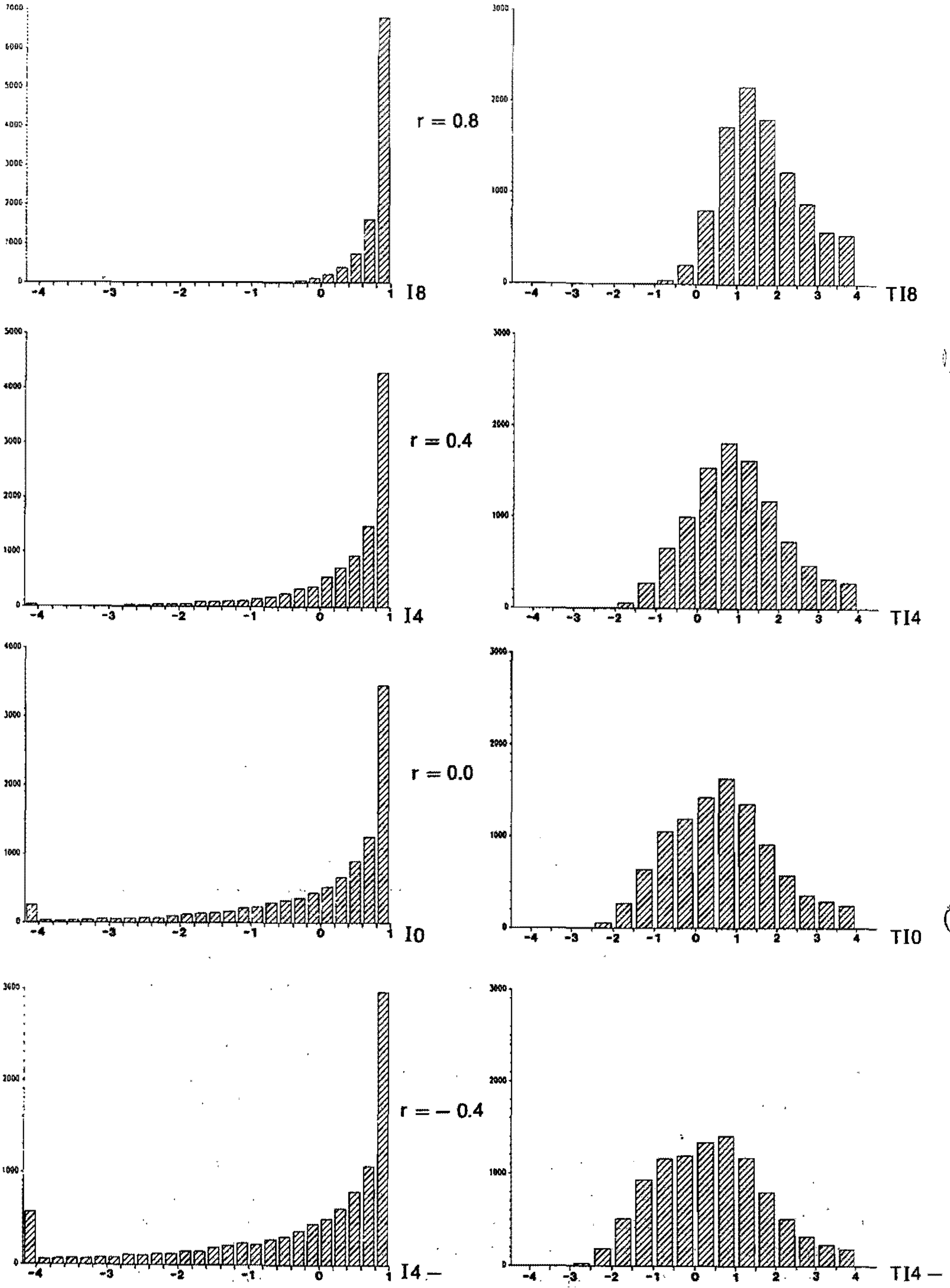
Table 1: Descriptive scores for the individual consistencies between bivariate nomal distributions with differing correlation (mean of 100 Monte-Carlo studies with $N=100$ in each case; standard deviations within the 100 studies in brackets)

	Correlation r				
Descriptive score	0.80	0.40	0.00	−0.40	−0.80
Mean of $I_{XY(r)}$	0.80	0.41	0.01	−0.37	−0.80
	(0.01)	(0.06)	(0.10)	(0.15)	(0.11)
Std of $I_{XY(r)}$	0.27	0.82	1.36	1.91	2.53
	(0.04)	(0.12)	(0.19)	(0.29)	(0.35)
$\sqrt{2} \cdot (1 - r)$	0.28	0.85	1.41	1.98	2.55
Skewness of $I_{XY(r)}$	−2.35	−2.51	−2.38	−2.45	−2.54
	(0.69)	(0.68)	(0.68)	(0.59)	(0.82)
Excess of $I_{XY(r)}$	7.25	8.21	7.36	7.63	8.66
	(5.63)	(5.65)	(5.40)	(4.66)	(7.49)
Mean of $TI_{XY(r)}$	1.63	1.00	0.65	0.41	0.21
	(0.06)	(0.09)	(0.11)	(0.11)	(0.10)
Std of $TI_{XY(r)}$	0.96	1.15	1.27	1.34	1.41
	(0.05)	(0.07)	(0.06)	(0.08)	(0.09)
Skewness of $TI_{XY(r)}$	0.41	0.25	0.26	0.26	0.29
	(0.17)	(0.18)	(0.17)	(0.15)	(0.17)
Excess of $TI_{XY(r)}$	−0.37	−0.19	−0.27	−0.28	−0.33
	(0.26)	(0.32)	(0.29)	(0.30)	(0.26)
Correlation $r_{XI_{XY}}$	0.00	0.02	0.01	0.02	0.01
	(0.11)	(0.11)	(0.13)	(0.15)	(0.15)
Correlation $r_{XTI_{XY}}$	0.00	0.01	0.02	0.01	0.01
	(0.11)	(0.09)	(0.09)	(0.08)	(0.07)

Figures 3 and 4 illustrate the nonlinear stochastic dependence of the (transformed) individual consistencies upon X. Figure 3 shows the most extreme dependence for $r_{XY} = -1$.

Figure 4 demonstrates for $-1 < r_{XY} < 1$ that this nonlinear stochastic dependence leads to a "hole" in the joint distribution of X, I_{XY}, or TI_{XY}, respectively, which increases the more negative the correlation r_{XY} becomes.

Figure 4 suggests that for the positive or – at worst – slightly negative correlations of studies of consistency, the nonlinear stochastic dependence between X and the individual consistencies or their transformations plays a minor role. Only extreme inconsistencies are affected insofar as they can only occur for extreme values of X. This relation must hold because


r = 0.8
I8
TI8
r = 0.4
I4
TI4
r = 0.0
I0
TI0
r = – 0.4
I4 –
TI4 –

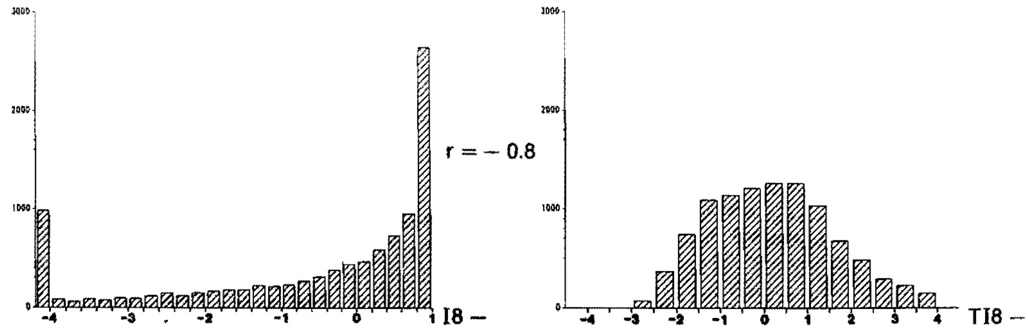


Figure 1: Individual consistencies (left), and their transformations (right), for bivariate normal distributions with varying correlation.

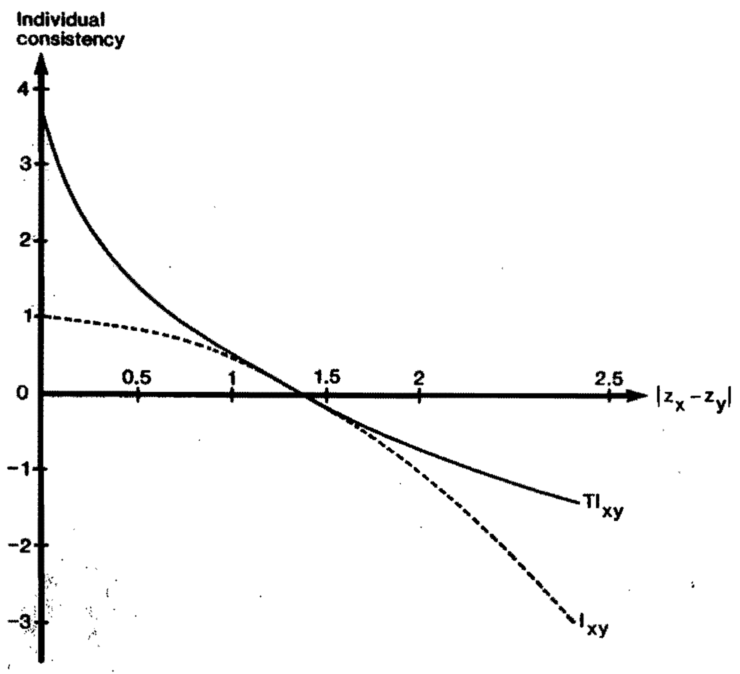


Figure 2: Dependence of the (transformed) individual consistencies upon the z-score difference.

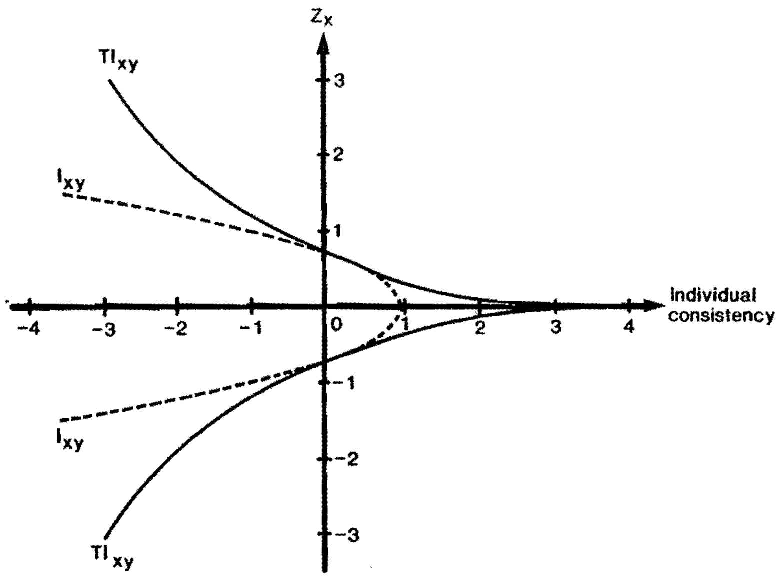
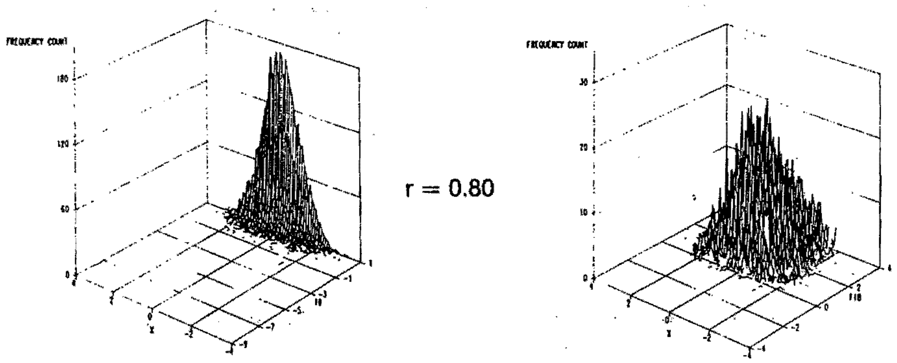


Figure 3: Perfect, nonlinear relation between initial scores and the (transformed) individual consistencies for $r_{XY} = -1$.

extreme inconsistencies can only be caused by switching from one extreme score to the opposite extreme score.

The increase of nonlinear stochastic dependence between X and the individual consistencies with increasingly negative correlations is an example of the sensitivity of the individual consistencies to sign changes of the variables compared. Note that in regression problems, no such sensitivity exists. As a consequence, studies of consistency should always try to score the variables compared for consistency in a way that minimizes negative correlations among these variables.



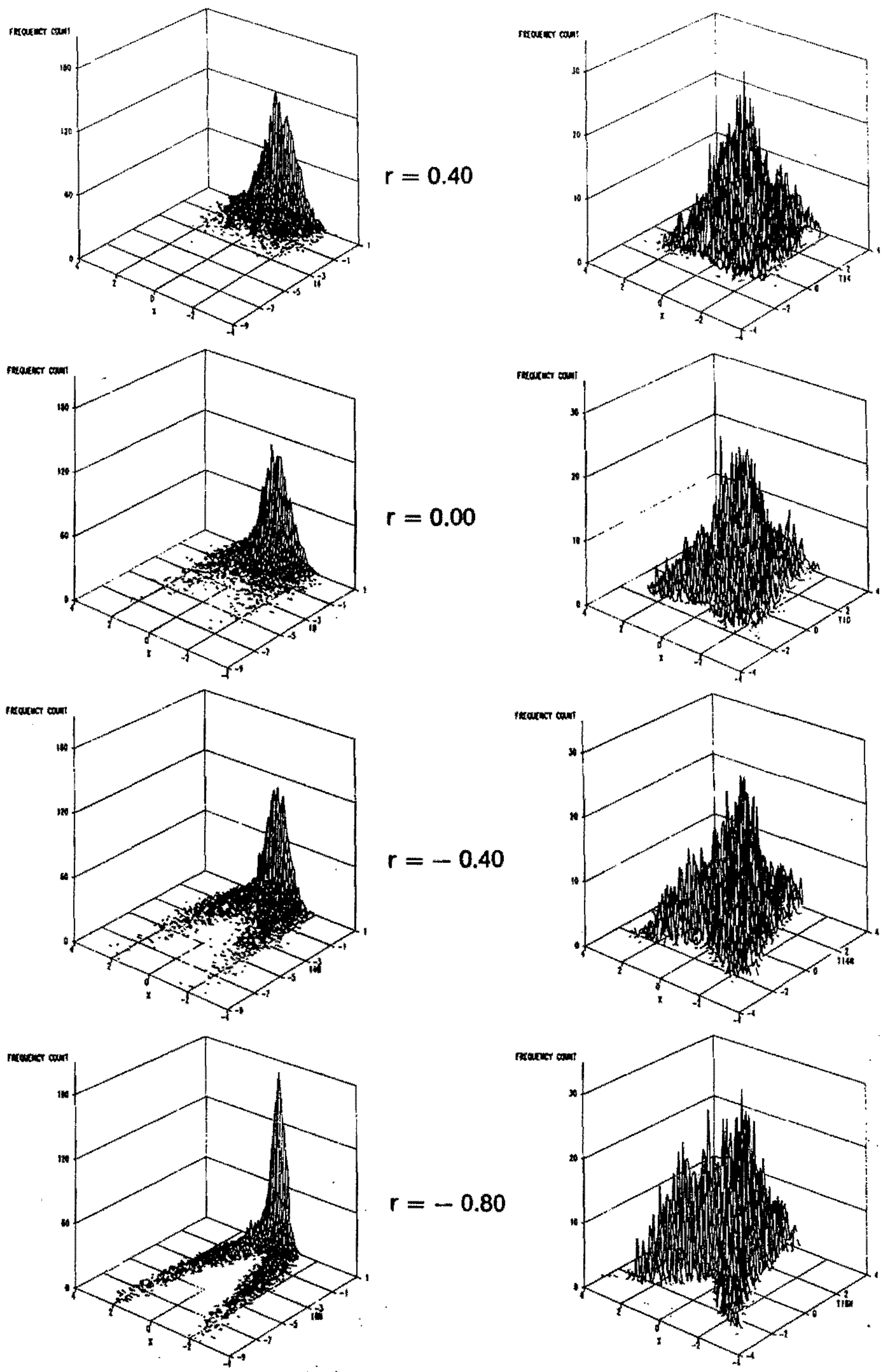


Figure 4: Results of the Monte-Carlo studies regarding the increasingly nonlinear stochastic dependence between X and the individual consistencies (left) and the transformed individual consistencies (right) for various correlations between X, Y (each diagram represents 100 studies with $N = 100$ each).

3. Example

The measurement of individual consistency is now illustrated with a data set reflecting the longitudinal stability of children's social competence (because in this example the consistency over time is investigated, I prefer the term (individual or differential) *stability*). The data stem from the Munich Longitudinal Study on the Genesis of Individual Competencies (LOGIC, Weinert & Schneider, 1986).

Figure 5 presents the scatterplot of the one-year stability of $r = 0.40$ of the observed rate of shy contact initiations among all initiations directed to peers during free-play in preschool for 68 children.

The scatterplot indicates that children's rate of shy behavior showed a substantial differential stability. This can be easily seen by comparing their scores with the *stability line* $y = (s_Y/s_X)(x - m_X) + m_Y$, where s_X, s_Y are the standard deviations of X, Y, and m_X, m_Y are the means of X, Y (cf. Fig. 5). The stability line is identical with the regression line for $r_{XY} = 1$.

The data presented in Figure 1 are now analyzed in terms of the coefficients of individual consistency. The distributions of the observed rates of shy behavior deviate somewhat from normality (for Kolmogorov's test for deviation from normality, $d = 0.15$, $p < 0.01$, for the first year of observation; $d = 0.08$, $p < 0.08$, for the second year). However, as Figure 6 indicates, these deviations are not extreme.

Figure 6 shows that the distribution of the (untransformed) individual consistencies I_{XY} is strongly skewed to the left with mean 0.40, standard

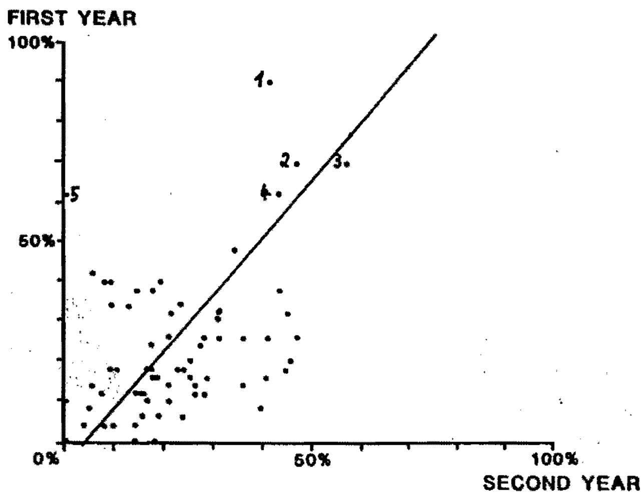


Figure 5: One-year stability of interindividual differences in the rate of shy behavior during contact initiation attempts at free-play in preschool ($n = 68$).

deviation 1.23, minimum − 7.49, and maximum 1.00; the distribution of the transformed individual consistencies TI_{XY} is fairly normal ($d = 0.07$, $p = 0.15$) with mean 1.16, standard deviation 1.31, minimum − 2.14, and maximum 3.80.

Comparing these data with the corresponding ones for bivariate normal distributions (cf. Table 1) indicates that both standard deviations and the mean of the transformed individual consistencies are somewhat greater than to be expected on the basis of a correlation of 0.40. Comparing Figures 1 and 6 suggests that this is due to the few children with very low or very high individual consistencies.

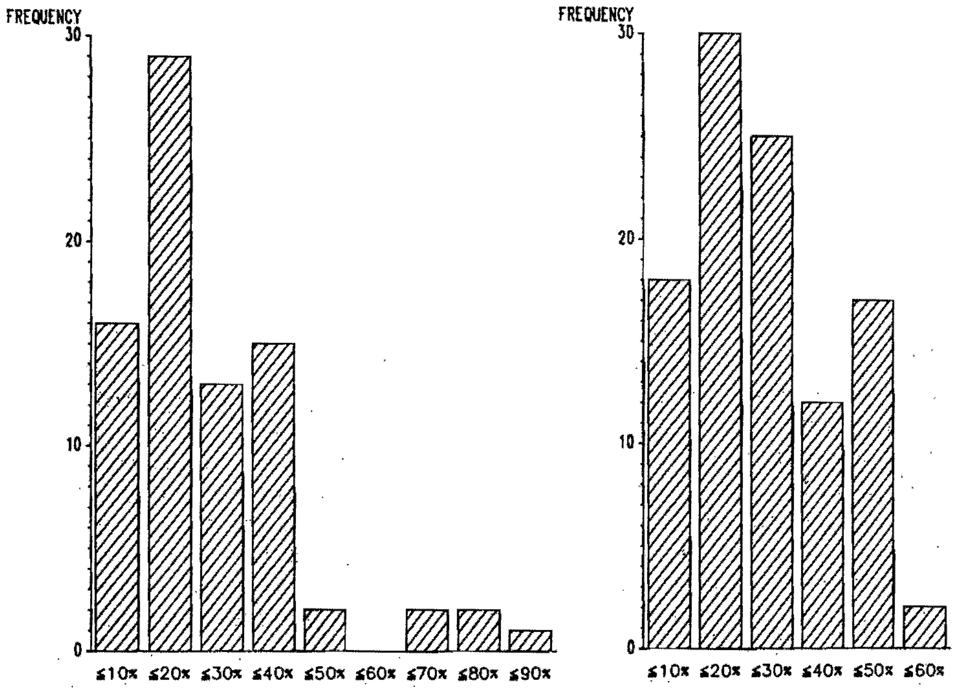
Table 2 contains some descriptive scores of the five children marked in Figure 5. Table 2 indicates that child no. 5 is extremely inconsistent in the rate of shy behavior observed. The other four children have a rate of shy behavior more than one standard deviation above the sample mean for both measurement points. Their individual consistencies still vary considerably, from the virtually perfect consistency of child no. 3 to a considerable inconsistency of child no. 1. Thus, the coefficients of individual consistency (particularly the untransformed scores) are sensitive to differences within extreme scores.

The dependence of the individual consistencies upon the scores of the two assessments compared was analyzed by correlating the transformed individual consistencies with these scores (the untransformed consistencies should not be used for these analyses because of their highly skewed distribution). A correlation of $r = -0.41$ ($p < 0.001$) was found for the first assessment, and one of $r = -0.12$ (ns) for the second assessment. Thus, the more children were found to be shy during the first observation period, the less consistent their rates of shy behavior were over the one-year period. For a closer analysis, the scatterplot of the correlation of $r = -0.41$ was investigated (cf. Figure 7).

The scatterplot indicates that the negative correlation between the scores of the first assessment and the individual consistencies strongly depended upon the two children with the lowest consistencies (no. 1 and 5 from Fig. 1, 7). Excluding these two children, the correlation decreases to $r = -0.28$ ($n = 66$, $p < 0.03$). On the other hand, the negative relation between the initial scores and the individual consistencies is not just an artifact. When the individual consistencies were correlated with nearly 100 different measures of the children's social behavior and cognitive functioning obtained during the first and the second year of assessment, a clearcut picture emerged. Only variables tapping children's shyness correlated at least $r = |0.30|$ with the individual consistencies for shy contact initiation behavior, and all of these correlations were negative. Besides observed shy contact initiations, the parental rating of children's shyness toward other children in the first year of assessment ($r = -0.39$), as well as in the second year ($r = -0.36$), the teacher rating "shy in the preschool group"

Table 2: Descriptive scores of the five children marked in figure five

	Child No.				
Score	1	2	3	4	5
z-score of 1st assessment z_1	3.92	2.80	2.78	2.32	2.32
z-score of 2nd assessment z_2	1.30	1.72	2.50	1.44	− 1.77
$\|z_1 - z_2\|$	2.62	1.08	0.28	0.88	4.09
Individual consistency	− 2.43	0.42	0.96	0.61	− 7.36
Transformed ind. consistency	− 1.23	0.45	1.93	0.71	− 2.12



Rate of shy behavior, 1st year

Rate of shy behavior, 2nd year

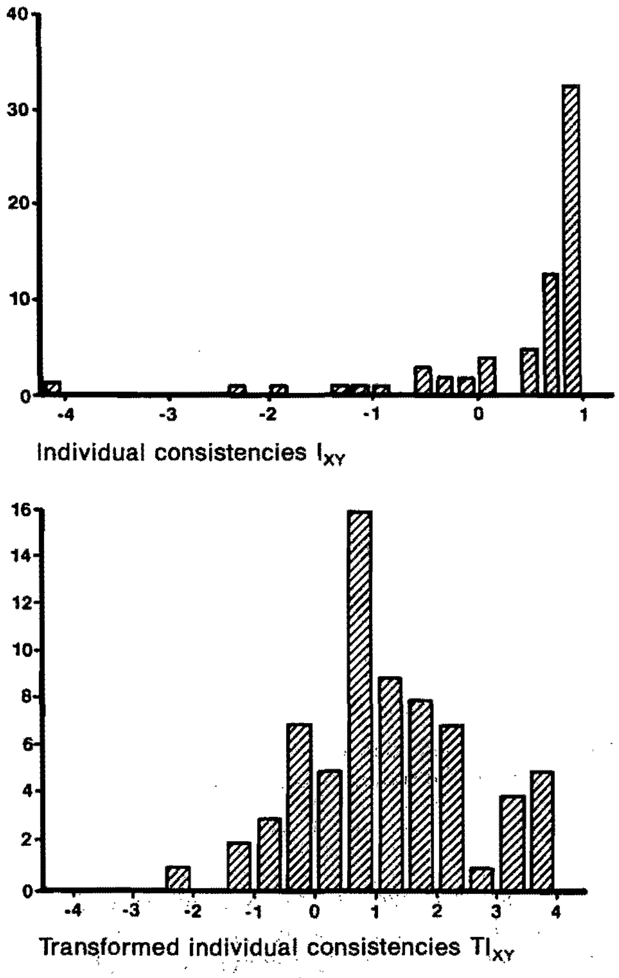


Figure 6: Distributions of the observed rates of shy behavior at the two assessments and of the (transformed) individual consistencies between these assessments.

($r = -0.30$), and the mean length of silences in children's conversations with an adult stranger ($r = -0.42$) showed substantial negative correlations with the individual consistencies (in each case, $p < 0.02$).

This correlational pattern supports the notion that the higher shyness is among young children, the less stable it is over time. This result fits with the data of the Harvard longitudinal study on behavioral inhibition (Reznick et al., 1986). In this study of extreme groups of shy and non-shy children, some of the children found to be very shy in the third year of life became much less shy two years later whereas no single instance of a

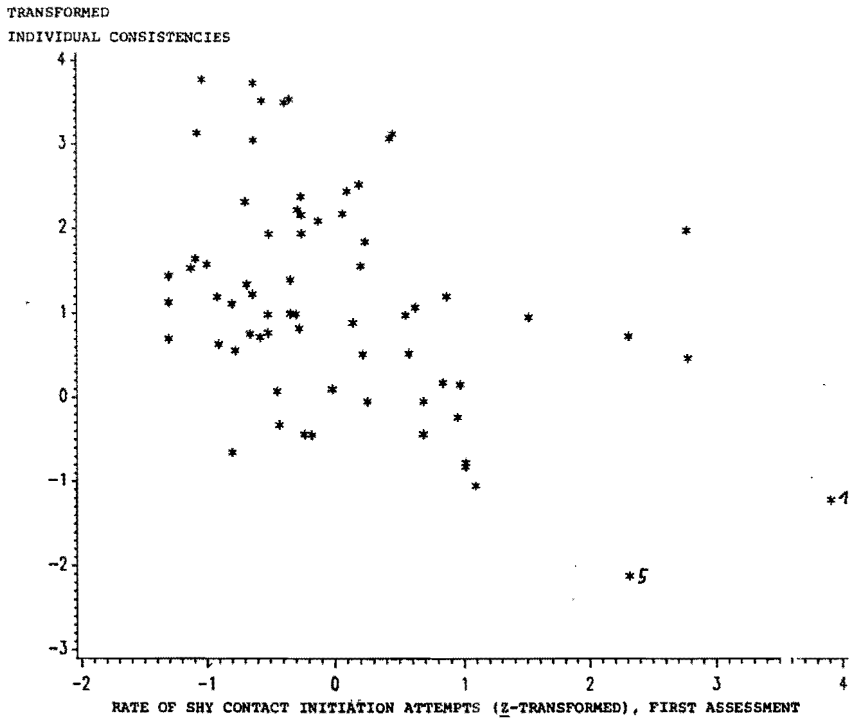


Figure 7: Scatterplot of the dependence of the transformed individual consistencies upon the scores of the first assessment.

change from low shyness to high shyness was observed during this two-year period. Thus, the differential consistency of shy behavior observed in the present study could be partly explained in a meaningful way by applying the proposed coefficients of individual consistency.

4. Discussion and conclusion

The example illustrates how the proposed coefficients of individual consistency can be applied to psychological data sets (cf. Asendorpf, in press, for other applications to longitudinal data). The coefficients of individual consistency may be also used to explore the consistency of interindividual differences in behavior across situations or different measures of the same construct.

The proposed coefficients of individual consistency fill a gap in present differential methodology by allowing a detailed descriptive analysis of differential consistency. Because the transformed individual consistencies are less sensitive to outliers than the raw individual consistencies, they provide a measure of consistency at the aggregate level which is more robust than the correlation (cf. Fig. 2 and Asendorpf, in press, for more evidence). Finally, the transformed individual consistencies open a new approach to studying moderator effects simply by correlating the individual consistencies with external variables.

The approach to the measurement of individual consistency presented here is based on a parametric approach. Alternatively, a non-parametric approach could be chosen by "individualizing" rank-order measures of consistency such as Kendall's τ. Although some information will be lost in this case, a non-parametric approach has the advantage of providing even more robust measures of (individual) consistency.

All in all, the measurement of individual consistency may open new avenues for the study of stability and change in human characteristics, and might help making differential psychology more differential than it is today.

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